April 2003 AH

1.	(a) Differentiate $y = 4^{x+1}$ (b) $y = \tan(x^2 + x + 2)$	**3,3**
2.	Use the substitution $u = 2x^2$ to find $\int \frac{x}{1+4x^2}dx$	**4**
3.	Find the term in x^3 in the expansion of $\left(3x + \frac{2}{x}\right)^5$.	**4**
4.	Express in partial fractions $\frac{2x+1}{(x+1)^2}$	**4**
5.	Find $\frac{dy}{dx}$ when $x^2 + xy + y^2 = 3$.	**4**
	Hence show that there are only 2 values of x where $\frac{dy}{dx} = 0$	**2**
6.	Given $z = x + iy$, find the equation of the locus of $\lvert z - i \rvert = \lvert z + i \rvert$	**4**
7.	For the system of equations $\begin{aligned} x - y + z &= 1 \\ x + y + 2z &= 0 \\ 2x - y + az &= 2 \end{aligned}$, use Gaussian elimination to express the value of z in terms of a. stating the value of a for which there is no solution. Hence write down the values of x,y and z when a = 3.	**5**
8.	Prove by induction that $\sum_{r=1}^{n} 2r^2 = \frac{1}{3}n(n+1)(2n+1)$	**5**

Over/

9. Given $A^2 = 5A - 3I$, where I is the Identity matrix,

find an expression for A^{-1} in terms of the matrices A and I. **4**
Find a similar expression for A^3.

10. (a) Find the Maclaurin series up to x^3 for e^x and $\sin x$ and write down the series for e^{-x}.

(b) Using the results of part (a) find the Maclaurin series up to terms in x^3 for $\dfrac{\sin x}{e^x}$ **3,3**

11. (a) Find the point of intersection of the line

L: $x = -t + 1$, $y = -t$, $z = t - 3$ and the plane P : $x - y + 2z = 9$

(b) Find the angle between the line L and the plane P. **3,3**

12. (a) Find values of x and y such that $29x + 17y = 1$

(b) Prove that $n^2 - n$ is never odd, where n is an integer. **4,2**

13./over

13. A function f is defined by $f(x)=\dfrac{x^2+x-2}{x+3}, x\neq -3$

(a) Find the coordinates of the intercepts with the x and y axes.
(b) Write down the equation of the vertical asymptote.
(c) Show that f has a non vertical asymptote and write down its equation.
(d) Find the coordinates of the stationary points and justify their nature.
(e) Sketch the graph of f showing all the main features.
(f) On the same diagram sketch the graph of $y=|f(x)|$ **2,1,3,4,1,1**

14. (a) Write down expressions in terms of n for

$$\sum_{k=1}^{n} 1,\quad \sum_{k=1}^{n} k,\quad \sum_{k=1}^{n} k^2$$

Hence find an expression in its simplest form for $\sum_{k=1}^{n}(3k^2-k-1)$

(b) Using a result from (a) find the value of $\sum_{k=11}^{20}(3k^2-k-1)$ **6,2**

15. The spread of a virus in a small village, population 400 is modelled by the differential equation $\dfrac{dV}{dt}=k(400-V)$ **,**
when $t=0$, $V=80$, where t is measured in weeks and V is the number of people with the virus at time t.

(a) Show that $\dfrac{1}{400-V}=Ae^{kt}$, stating the exact value of A

Hence express V explicitly in terms of t. **6**

(b) Given after 7 weeks the number of people with the virus has doubled, find the value of k, correct to 2 significant figures.
2

(c) The spread of the virus is described as an epidemic if more than $\dfrac{2}{3}$ of the population are affected after 20 weeks, was this an epidemic?
(justify your answer) **2**

16. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{-3x},$$

hence find the particular solution when both
$y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$. **7,3**

End of paper